Linear Precoding and Iterative LMMSE Detection for MIMO Channels with and without CSIT

Xiaojun Yuan

Research Fellow

EE Dept., City University of Hong Kong

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Related Work

- Xiaojun Yuan and Li Ping, "Space-time linear precoding and iterative LMMSE detection for MIMO systems without CSIT," submitted to ISIT 2011.
- Xiaojun Yuan, Li Ping, and Aleksandar Kavcic, "Achievable rates of MIMO systems with linear precoding and iterative linear MMSE detection," submitted to ISIT 2011.
- Xiaojun Yuan, Li Ping, and Aleksandar Kavcic, "Achievable rates of coded MIMO systems with linear precoding and iterative linear MMSE detection," submitted to *IEEE Trans. Inform. Theory*.

- Prof. Li Ping is with the Department of Electronic Engineering, City University of Hong Kong, HK SAR.
- Prof. Aleksandar Kavcic is with the Department of Electrical Engineering, University of Hawaii at Manoa, Honolulu, USA.

Outline

- Background
- Linear Precoding with Perfect CSIT
- Linear Precoding without CSIT
- Conclusions

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  - Iterative LMMSE Detection
  - Area Theorems
- Linear Precoding with Perfect CSIT
- Linear Precoding without CSIT
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Iterative LMMSE Detection

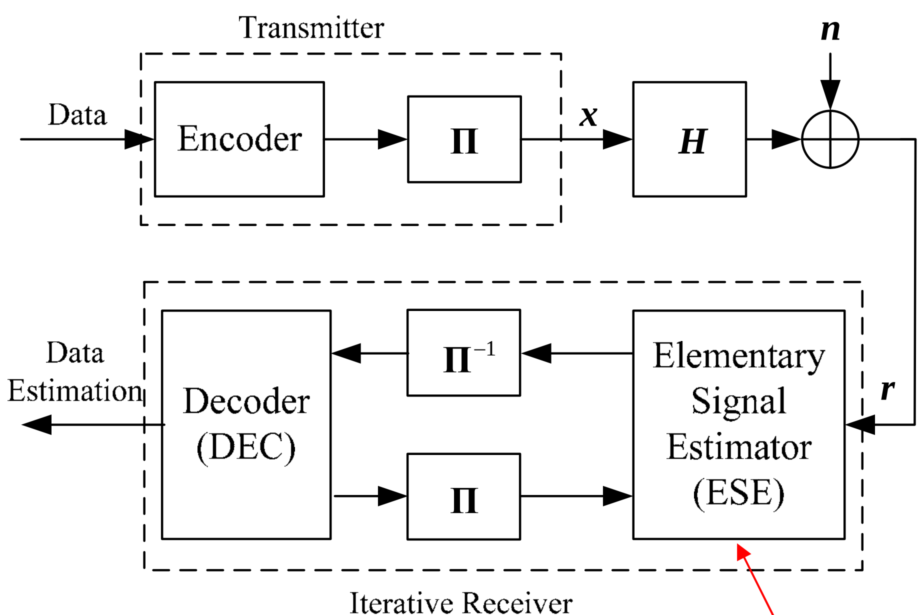
- Iterative LMMSE detection is an efficient interference cancellation technique for large linear systems. It has been discussed extensively in the following papers.

X. Wang and H. V. Poor, "Iterative (turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Trans. Commun.*, vol. 47, pp. 1046–1061, July 1999.

M. Tüchler, R. Koetter, and A. Singer, "Turbo equalization: principles and new results," *IEEE Trans. Commun.,* vol. 50, pp. 754-767, May 2002.

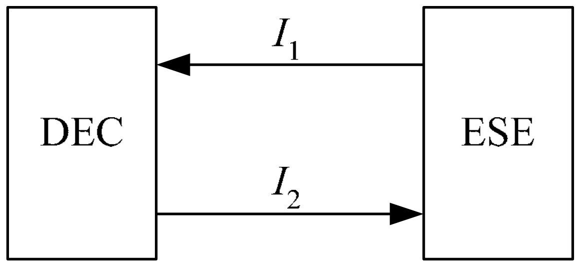
X. Yuan, Q. Guo, X. Wang and Li Ping, "Evolution analysis of low-cost iterative equalization in coded linear systems with cyclic prefixes," *IEEE Journal of Select. Areas in Communications,* February 2008.

Iterative LMMSE Detection

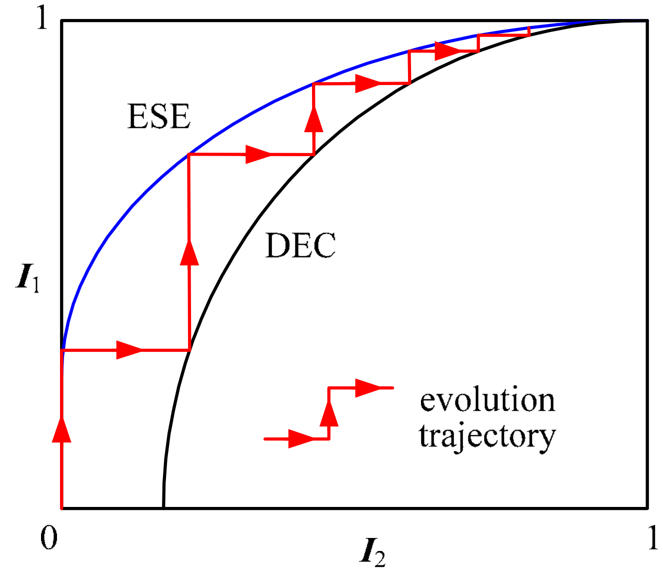


The ESE performs LMMSE estimation.

EXIT Chart Analysis



iterative receiver



EXIT chart

It is well known that curve-matching can lead to good performance.

Curve matching → Capacity achieving ?

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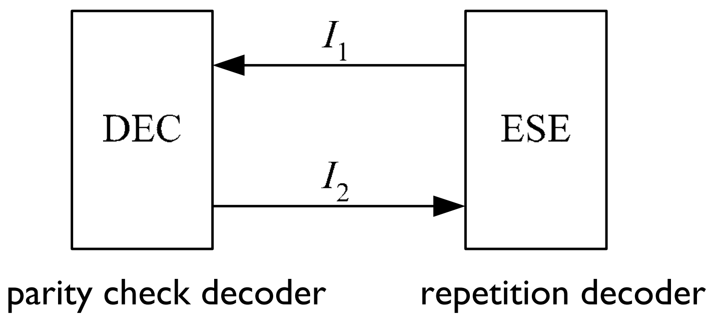
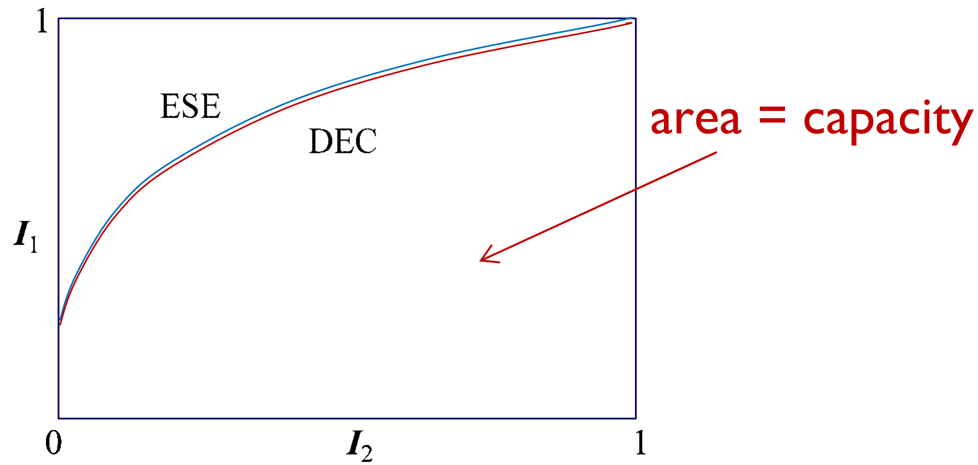
Binary Erasure Channel





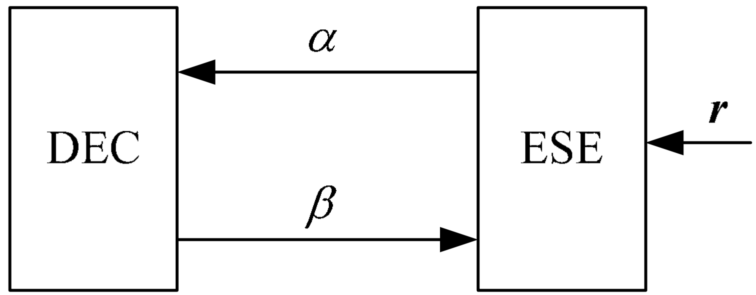
For a binary erasure channel, it is shown in the following paper that

LDPC codes with iterative decoding can achieve capacity

if the two EXIT curves are matched.

A. Ashikhmin, G. Kramer, and S. ten Brink, "Extrinsic information transfer functions: Model and erasure channel properties," *IEEE Trans. Inf. Theory*, vol. 50, no. 11, pp. 2657–2673, Nov. 2004.

Binary-Input AWGN Channel



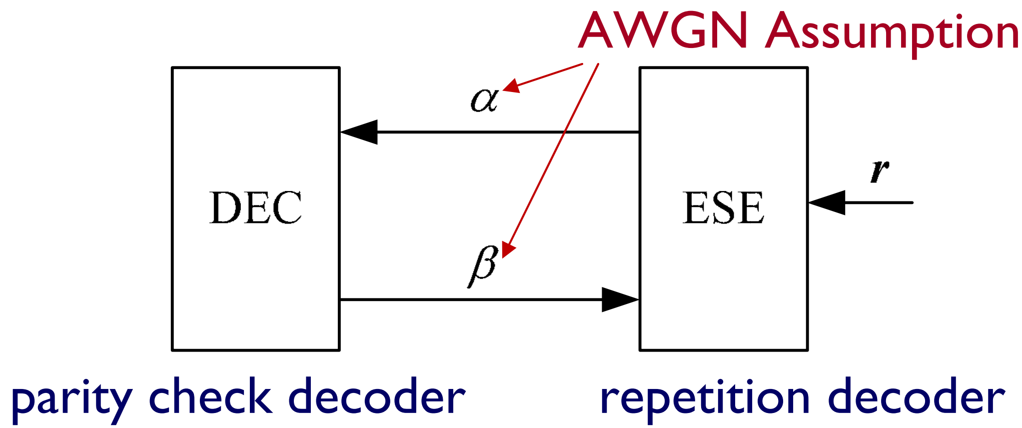
parity check decoder repetition decoder

BI-AWGN channel capacity can be achieved using LDPC codes under the assumptions that

(i) the ESE and DEC are locally optimal,
(ii) the messages in α and β are observations from an AWGN channel,
(iii) the ESE and DEC are matched.

K. Bhattad and K. R. Narayanan, "An MSE-based transfer chart for analyzing iterative decoding schemes using a Gaussian approximation," *IEEE Trans. Inform. Theory*, vol. 53, no. 1, Jan. 2007.

Binary-Input AWGN Channel



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Extension to MIMO Channels

- Challenges
  - How to handle a MIMO channel? The AWGN assumption generally does not hold for such a channel.
  - How to handle non-binary signaling? The AWGN assumption generally does not hold for such signaling.
  - How to achieve the MIMO channel capacity with/without CSIT?

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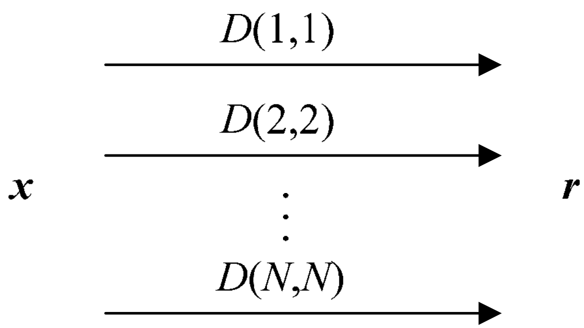
SVD with Perfect CSIT

- Singular Value Decomposition (SVD) of the MIMO Channel

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^{\mathrm{H}}$$

- With perfect channel state information (CSI), $\boldsymbol{U}$ and $\boldsymbol{V}$ can be canceled.
- Equivalent Parallel Channels

$$\boldsymbol{r} = \boldsymbol{D}\boldsymbol{x} + \boldsymbol{n}$$

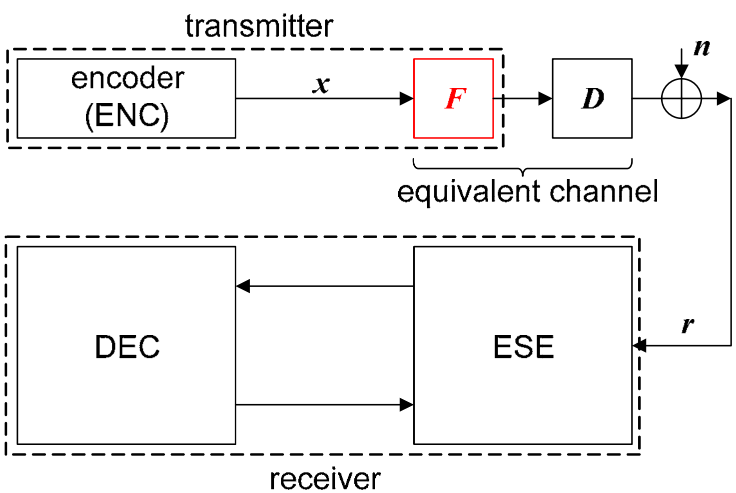


Linear Precoding (LP) and ILMMSE Detection

- Equivalent Channel Model

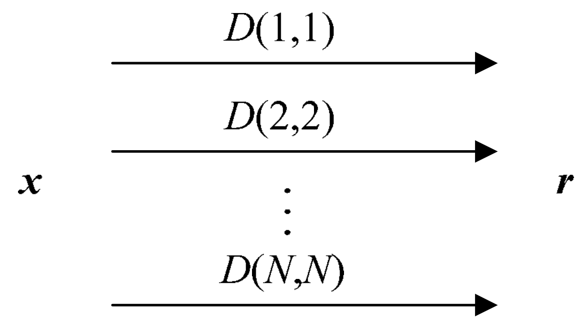
$$\boldsymbol{r} = \boldsymbol{D}\boldsymbol{F}\boldsymbol{x} + \boldsymbol{n}$$

- Dispersion Matrix $\boldsymbol{F}$: uniformly spread the symbols in $\boldsymbol{x}$ over the parallel channels
- Possible choices of $\boldsymbol{F}$
  - Discrete Fourier transform (DFT) matrix
  - Hadamard matrix
- Iterative detection is necessary since $\boldsymbol{F}$ introduces severe ISI.



Compared with other Transmission Strategies over Parallel Channels

- Multi-code transmission
  - Large transmission delay
  - Sensitive to CSIT error
- Single-code direct transmission
  - Performance loss due to channel variation
- Single-code with rate allocation (adaptive modulation)
  - Performance loss due to quantization
  - Joint design of adaptive modulation and the FEC code is a difficult problem
  - Not robust to CSIT error



parallel channels

- Single-code with linear precoding can avoid the above disadvantages

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- Linear Precoding with Perfect CSIT
  - AWGN Assumption for the ESE output
  - AWGN Assumption for the DEC output
  - Area Theorem
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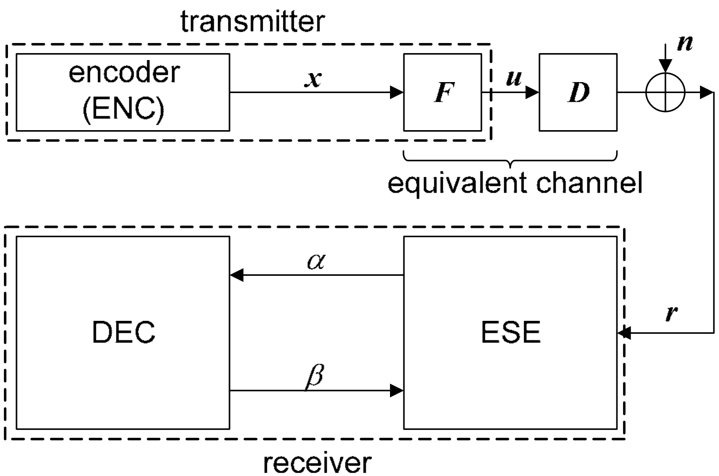
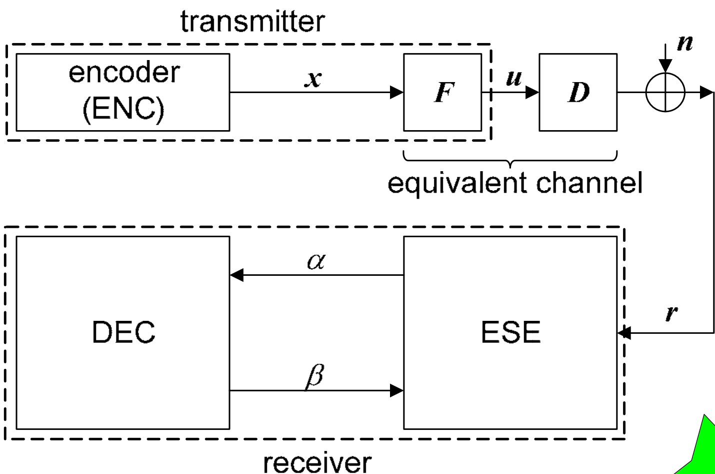
Property of the ESE Output



The ESE estimates $\boldsymbol{x}$ based on the channel observation $\boldsymbol{r}$ and the message β.

Property of the ESE Output



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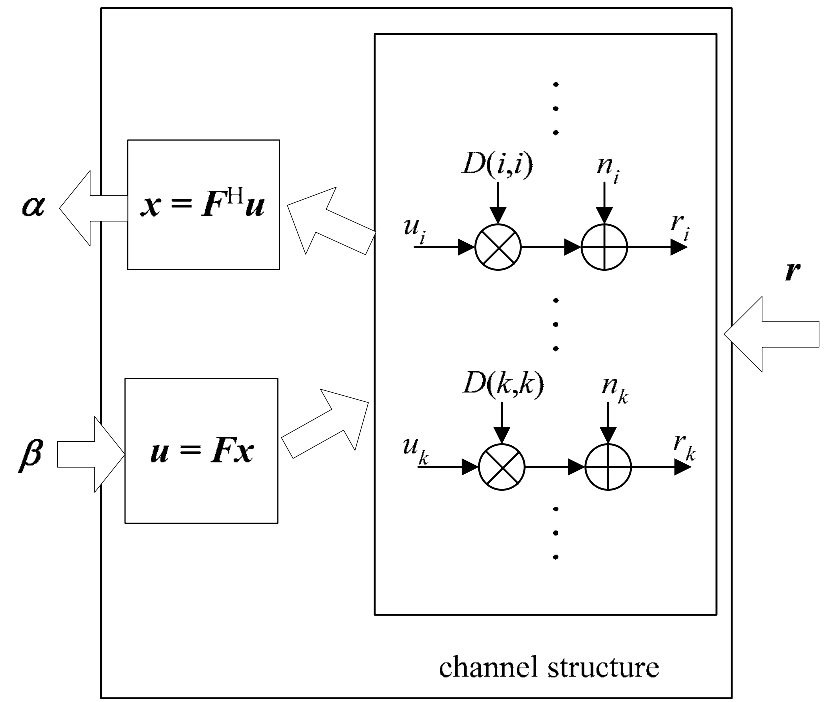

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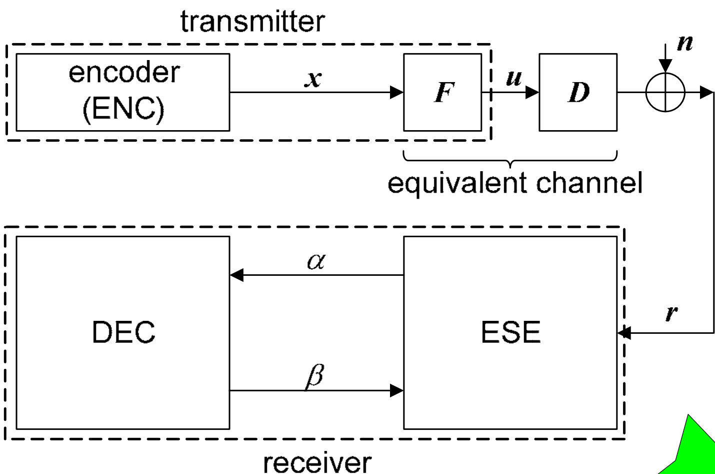
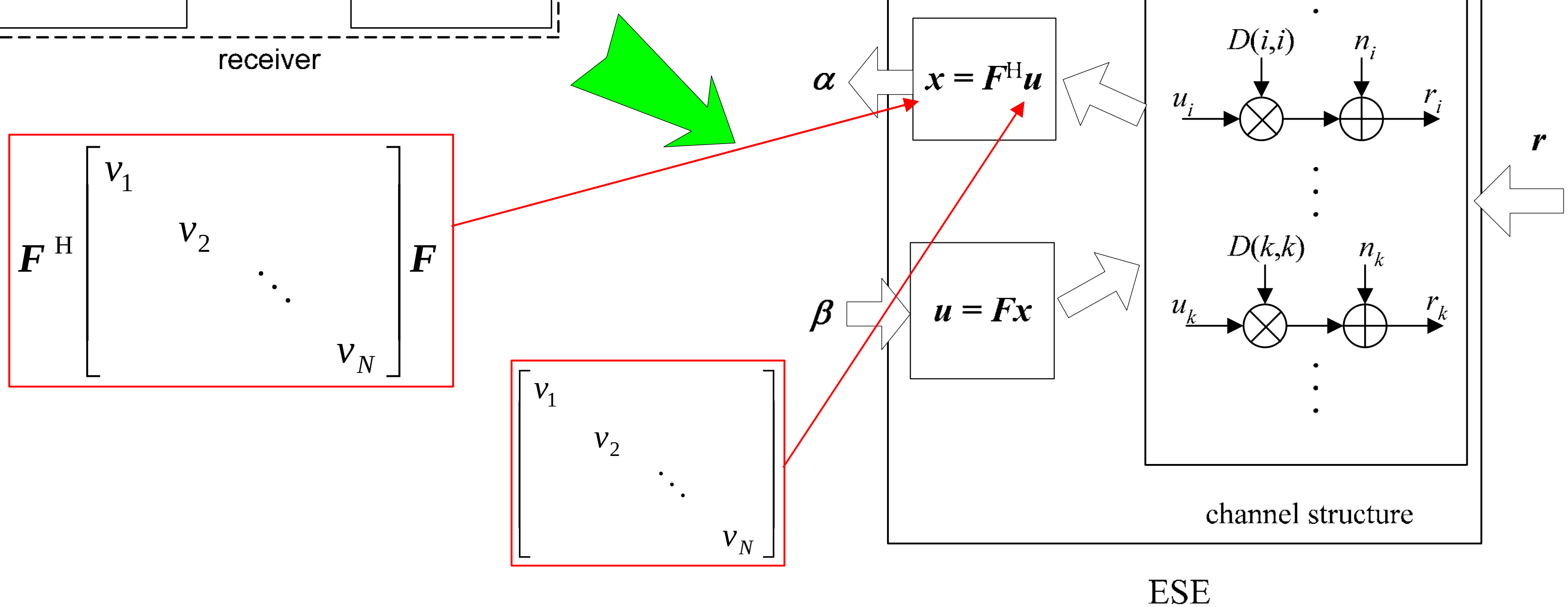




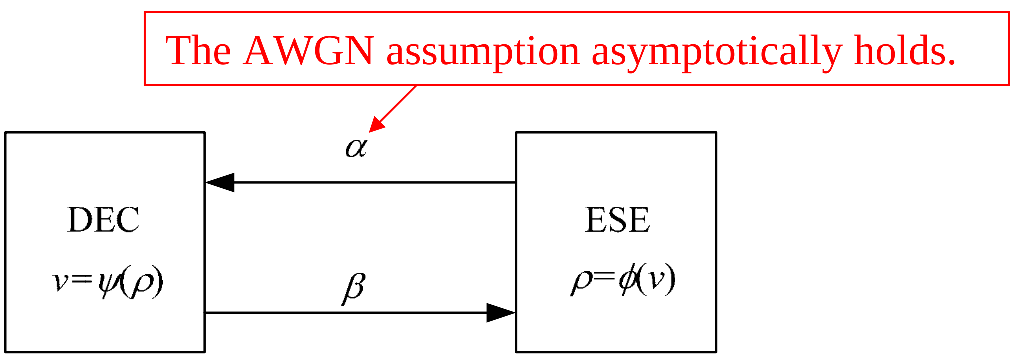
Asymptotic AWGN Property of the ESE Output

- **Theorem 1:** The ESE follows the LMMSE estimation. Then, the ESE output is

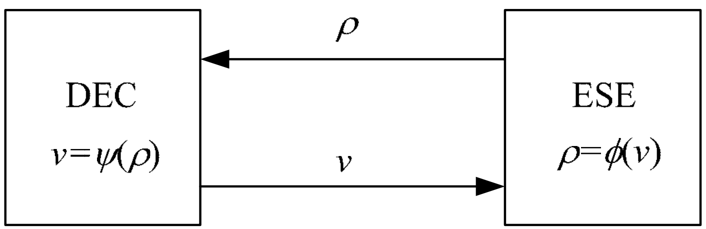
$$\alpha_i = x_i + w_i$$

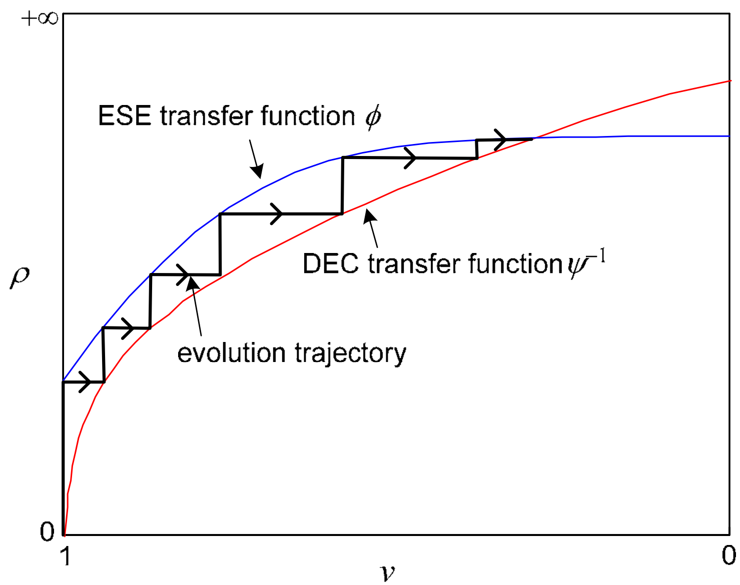
where w_i is independent of x_i and, as the DFT length tends to infinity, w_i converges in distribution to $\mathcal{CN}(0, 1/\rho)$ with

$$\rho = \phi(v) = \left(\frac{1}{N} \sum_{k=0}^{N-1} \left(\frac{1}{v} + \frac{|D(k,k)|^2}{\sigma^2} \right)^{-1} \right)^{-1} - \frac{1}{v}.$$



SNR-Variance Transfer (SVT) Chart





- Convergence Point: $\rho^* = \phi(\psi(\rho^*))$ or equivalently, $\phi(v^*) = \psi^{-1}(v^*)$
- Error Free Decoding: $v^* = 0$

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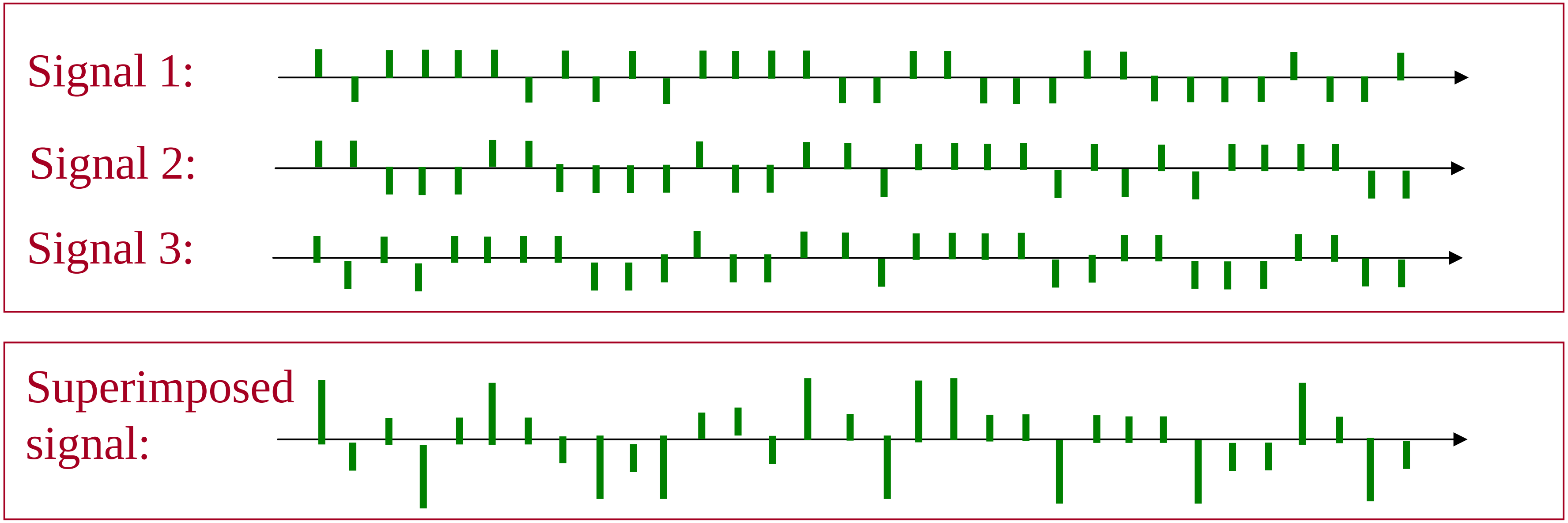
SCM Signaling is Nearly Gaussian



We can see that the signal distribution of SCM is approximately Gaussian. Intuitively, the corresponding DEC output is approximately AWGN.

Maximum Code Rate Given $\psi(\rho)$

- **Theorem 2:** Assume that a function $\psi(\rho)$ is first-order differentiable and monotonically decreasing in ρ, and that

$$R = \int_0^\infty \frac{1}{\psi(\rho)^{-1} + \rho} \mathrm{d}\rho < \infty$$

  Let $\psi(\rho)$ be the SNR-variance transfer function of the code used in the DEC. Then, the maximum rate of the code (per entry of $\boldsymbol{x}$) is given by R above.

- Comment: It can be shown that the above rate is achievable by assuming that the output messages of the DEC are AWGN. This assumption can be asymptotically ensured by superposition coded modulation (SCM).

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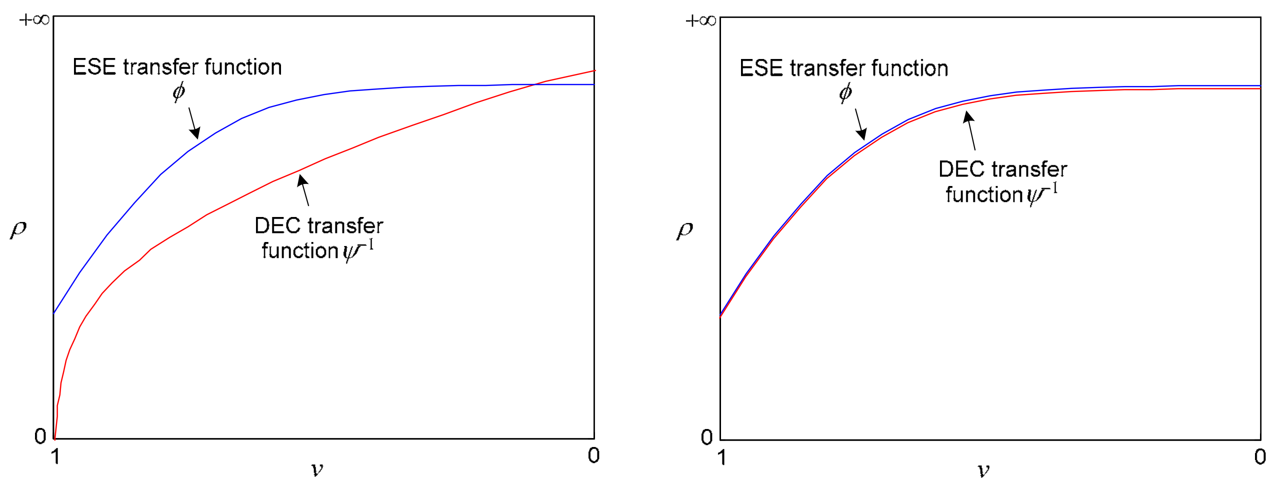
What is the maximum achievable rate of the proposed LP-ILMMSE scheme?

Curve-Matching Principle

- The ESE and DEC are **matched** if

$$\psi(\rho) = \phi^{-1}(\rho), \qquad \text{for any } \rho \in [0, +\infty).$$



not matched matched

Achieving iid MIMO Capacity

- **Theorem 3:** If the ESE and DEC are matched, the maximum achievable rate of the LP-ILMMSE scheme is given by

$$R = \log\left|\boldsymbol{I} + \boldsymbol{H}\boldsymbol{H}^{\mathrm{H}}\right|.$$

- **Sketch of Proof:**

$$R = \int_0^\infty \frac{1}{\psi(\rho)^{-1} + \rho}\,\mathrm{d}\rho \qquad \text{.................................... from Theorem 2}$$

$$= \int_0^\infty \frac{1}{(\phi^{-1}(\rho))^{-1} + \rho}\,\mathrm{d}\rho \qquad \text{.............................. from curve-matching}$$

$$= \ldots\ldots$$

$$\phi(v) = \left(\frac{1}{N}\sum_{k=0}^{N-1}\left(\frac{1}{v} + \frac{|D(k,k)|^2}{\sigma^2}\right)^{-1}\right)^{-1} - \frac{1}{v} \qquad \text{.............. from Theorem 1}$$

Theorem 3 says that the LP-ILMMSE scheme can achieve the iid MIMO capacity.

How to surpass the iid capacity?

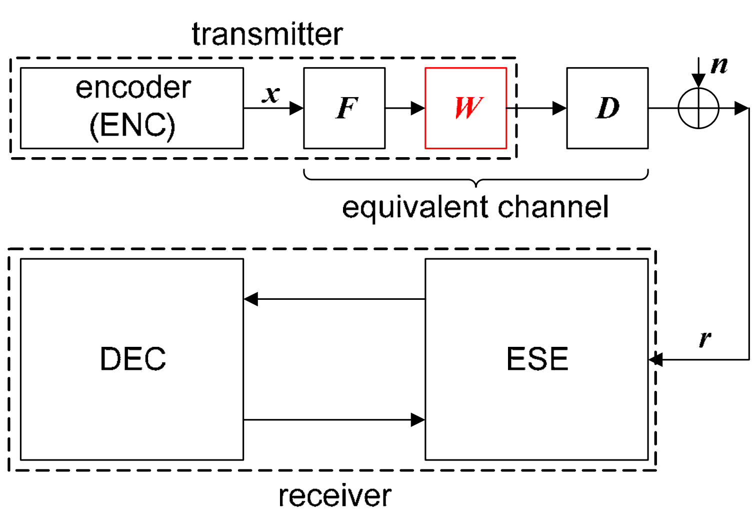
Power Allocation Precoder W

- Equivalent Channel Model after Linear Precoding

$$\boldsymbol{r} = \boldsymbol{DWFx} + \boldsymbol{n}$$

- Power Allocation Matrix W: allocate power over the parallel channels

- It can be proven that, if W follows the water-filling principle, the iterative scheme can achieve the water-filling capacity of the MIMO channel.



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Practical LP-ILMMSE System Design

- If the ESE and DEC are matched, a properly designed LP-ILMMSE scheme with SCM can asymptotically achieve the water-filling channel capacity as the number of SCM layers tends to infinity.
- In practice, due to complexity consideration, the number of SCM layers cannot be too large.
- We next consider the LP-ILMMSE scheme with a finite number of SCM layers.

LP-ILMMSE with Finite SCM Layers

- System Settings
  - A randomly generated 2-by-2 MIMO 3-tap ISI channel

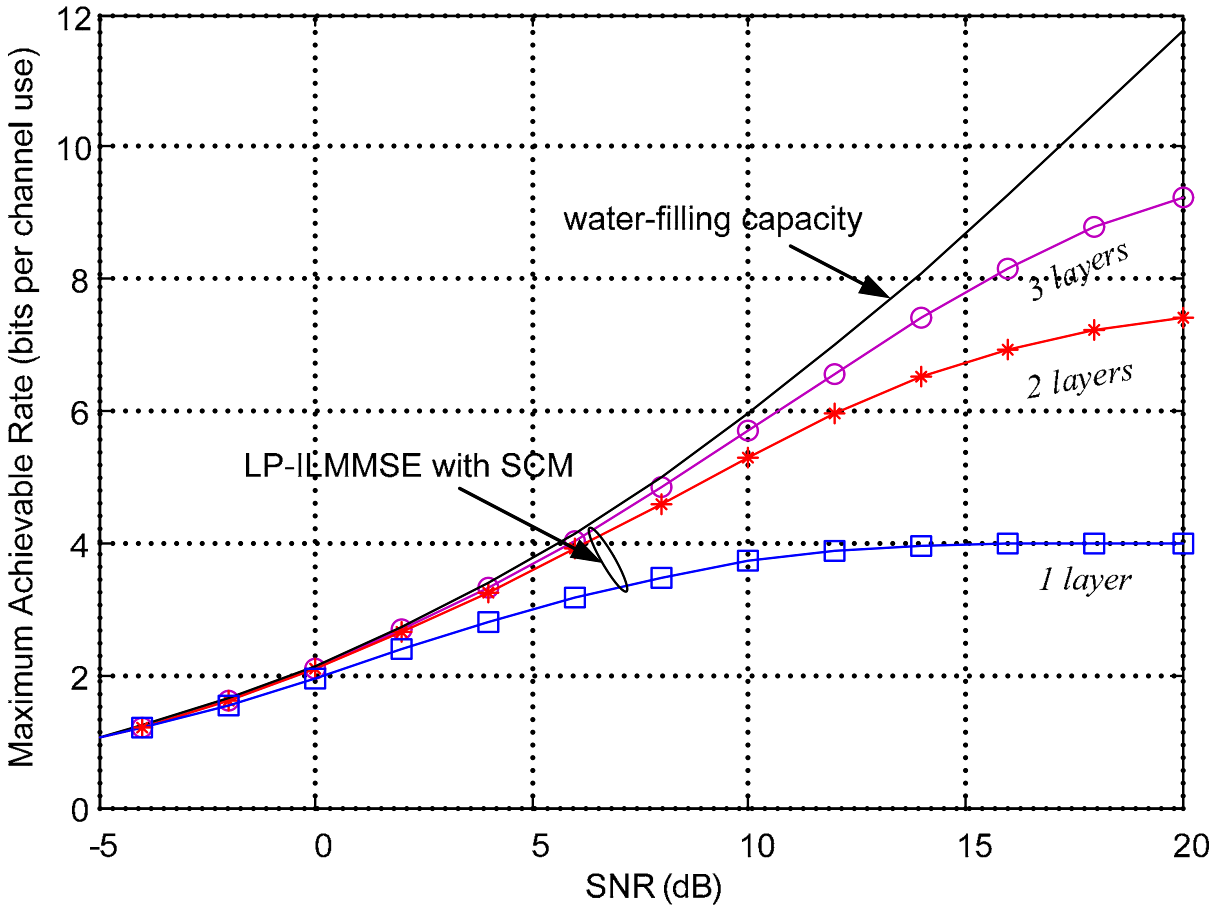
$$\begin{bmatrix} 0.5339 + j0.5395 & -0.4245 + j0.0648 \\ -0.3347 - j0.3727 & -0.4672 - j0.2420 \end{bmatrix}$$

$$\begin{bmatrix} 0.0582 - j0.2706 & 0.1525 - j0.7565 \\ -0.4968 - j0.1543 & -0.5243 - j0.5915 \end{bmatrix}$$

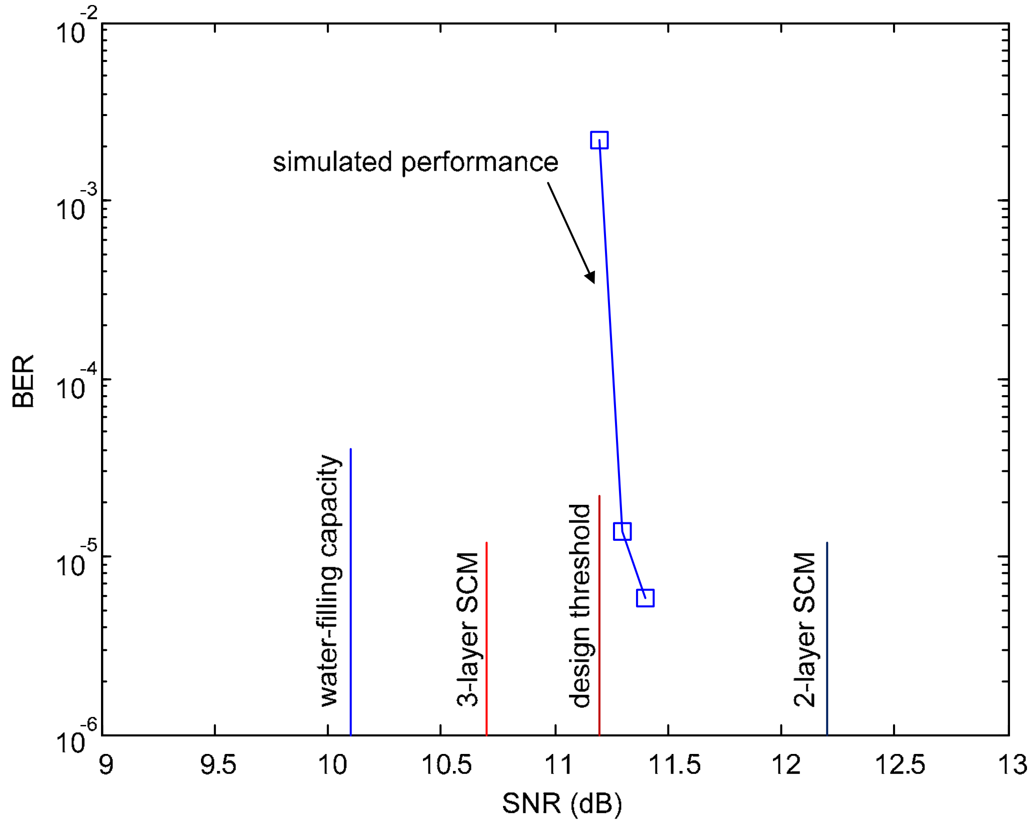
$$\begin{bmatrix} -0.5262 - j0.2654 & -0.3714 - j0.2865 \\ 0.6721 - j0.1635 & 0.1607 - j0.2695 \end{bmatrix}$$

  - LP-ILMMSE scheme with SCM
  - DFT length = 256
  - SCM with each layer QPSK modulated
    - Power allocation for the 2-Layer Case: $p_1 = 0.79$ and $p_2 = 0.21$
    - Power allocation for the 3-Layer Case: $p_1 = 0.61$, $p_2 = 0.295$, and $p_3 = 0.095$

Achievable Rates of LP-ILMMSE with Constrained Signaling: 2X2 MIMO



Three-Layer SCM with Matched LDPC Codes: Simulated Performance



- LP-ILMMSE scheme with 3-layer SCM over the randomly generated MIMO ISI channel
- Each layer use an irregular LDPC code followed by QPSK mapping
- System throughput = 6 bits per channel use. Code length = 10^5

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Linear Precoding without CSIT

- Singular Value Decomposition (SVD) of the MIMO Channel

$$\boldsymbol{H} = \boldsymbol{U}\boldsymbol{D}\boldsymbol{V}^{\mathrm{H}}$$

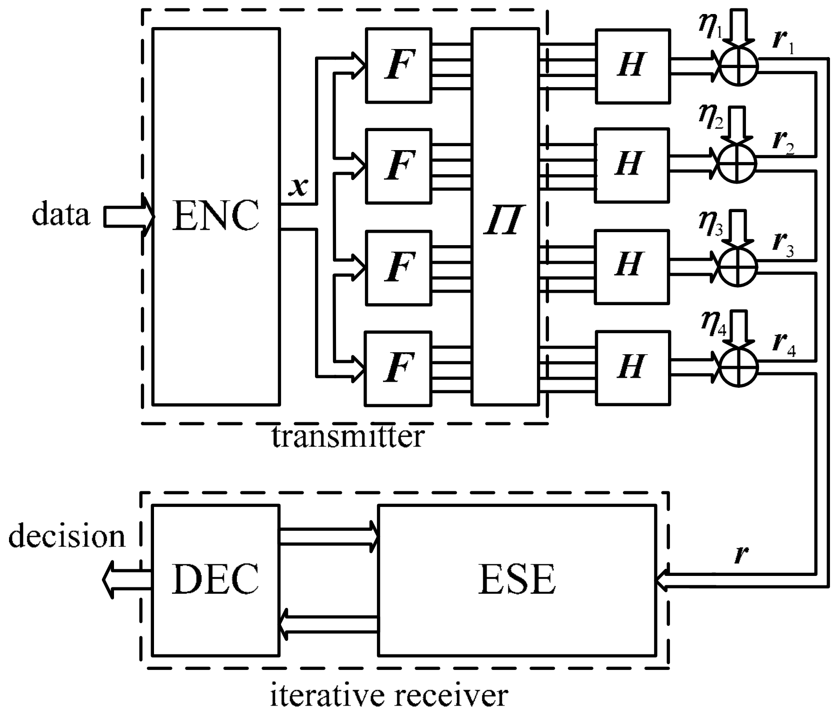
- If the channel is not known, $\boldsymbol{V}$ cannot be canceled at the transmitter side. Thus, the aforementioned linear precoding (LP) technique fails.

- Challenge

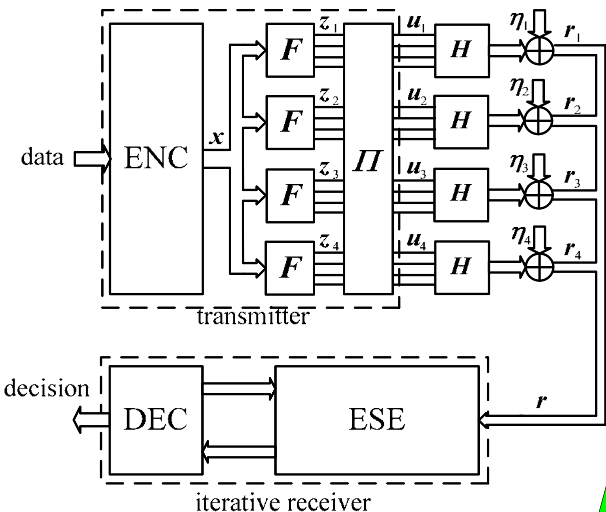
  Design a linear precoding system such that our established area theorem is still applicable

Space-Time Linear Precoding

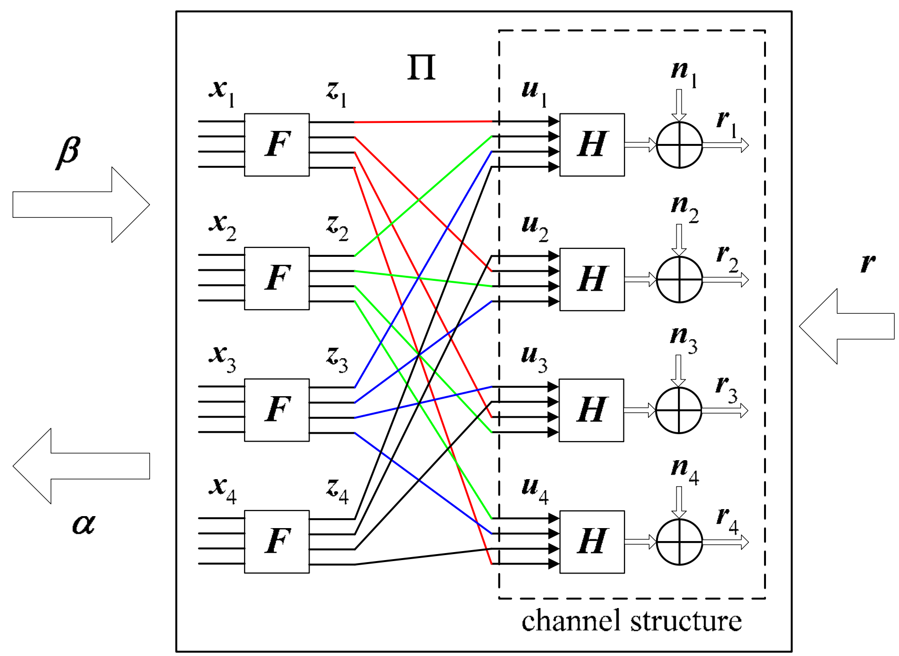
- Slow-Fading Assumption: The channel remains unchanged for at least N channel uses, where N is the number of transmit antennas.



Permutation Matrix Π



- Design Criteria of Permutation Matrix Π
  - No two entries of z_i for each index i are transmitted at a same channel use; and
  - No two entries of z_i are transmitted at a same antenna.



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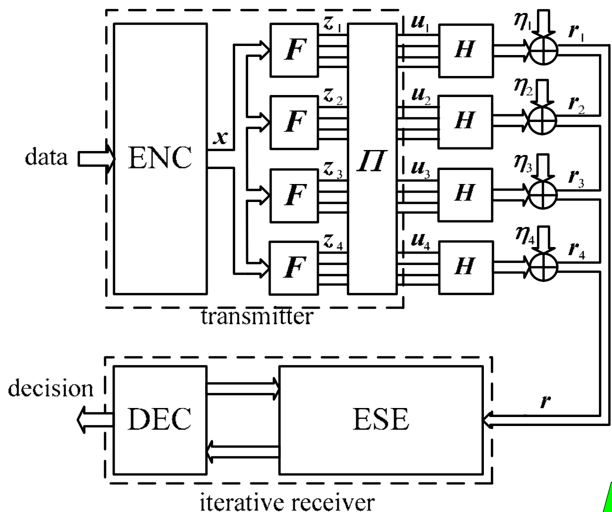

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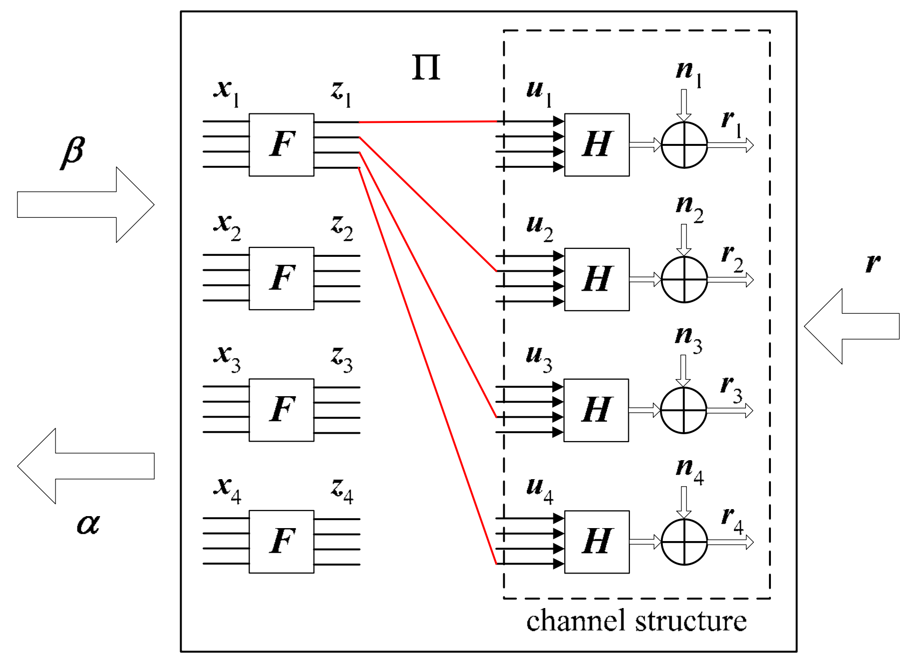

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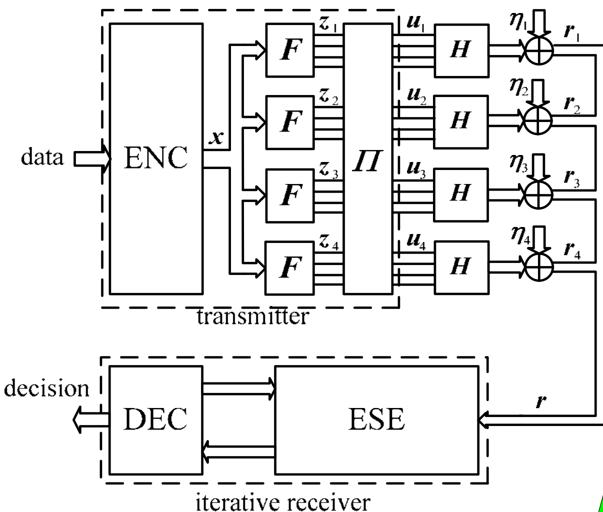

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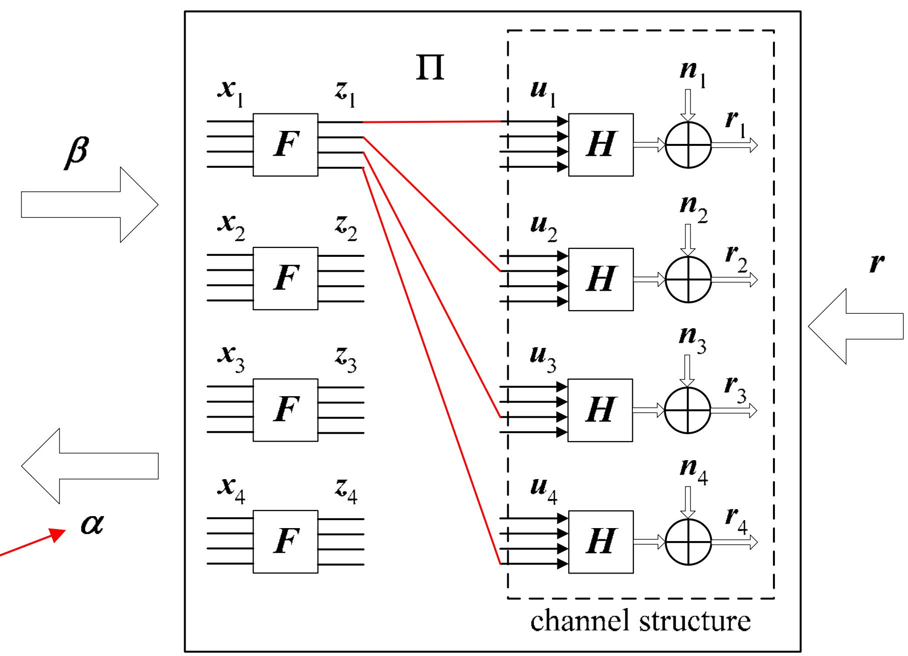


ESE

The AWGN assumption holds, and so is the area theorem.

LP-ILMMSE over Fading MIMO Channels without CSIT

- Ergodic case: A transmission block sees many independent channel realizations
  - The ESE transfer function $\phi(v)$ is a deterministic function
  - Perfect curve-matching is possible even without knowing CSIT

$$\phi(v) = \left(\frac{1}{N} \sum_{k=0}^{N-1} \left(\frac{1}{v} + \frac{|D(k,k)|^2}{\sigma^2} \right)^{-1} \right)^{-1} - \frac{1}{v}$$

- Outage case: A transmission block only sees one or a few independent channel realizations
  - $\phi(v)$ is a random function
  - Perfect curve-matching is impossible. Instead, we can design a fixed code that statistically best matches the ESE transfer curves

Design of DEC Transfer Function in the Outage Case

- **Problem**:

  Given a predetermined outage probability ε, what is the maximum achievable rate of the LP-ILMMSE scheme?

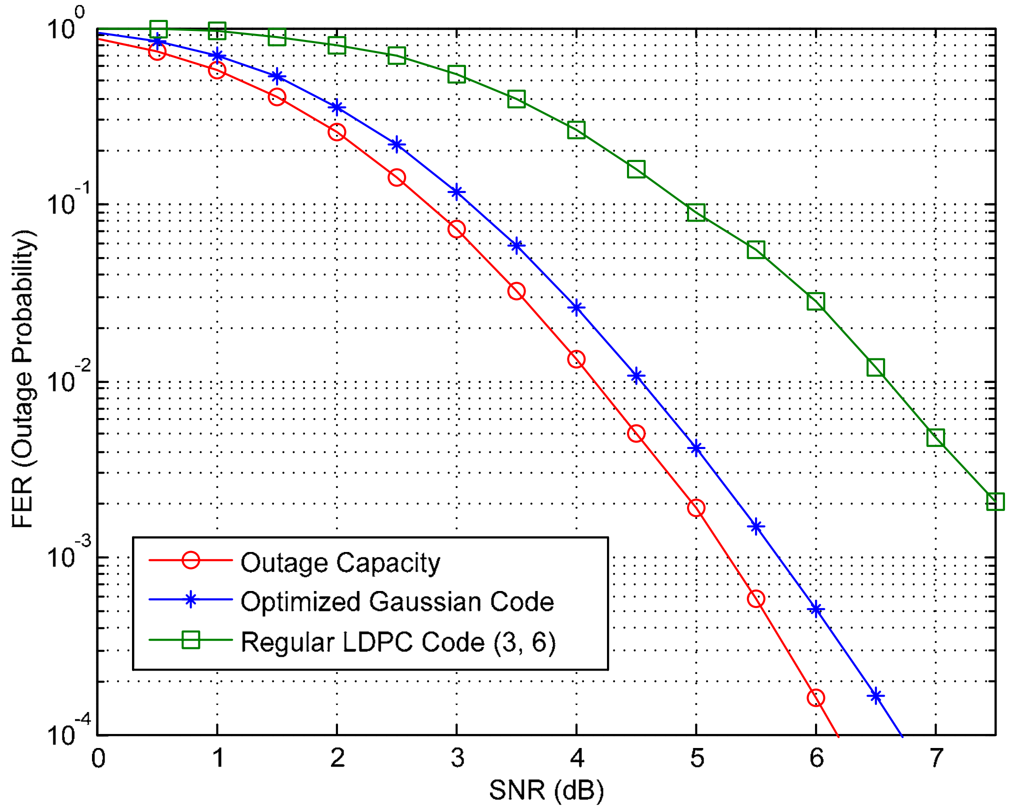
- **Problem Formulation**

$$\underset{\psi(\rho)}{\text{maximize}} \quad R = \int_0^\infty \frac{1}{\psi(\rho)^{-1} + \rho} \mathrm{d}\rho$$

$$\text{subject to} \quad \text{outage probability} \le \text{predetermined } \varepsilon$$

- Exact solution to the above problem is difficult. Some approximations can be introduced to obtain a reasonably good solution.

Numerical Results



The outage capacity and the frame error rate of the optimized LP-ILMMSE scheme in the quasi-static Rayleigh-fading 4x4 MIMO channel. System throughput = 4 bits per channel use.

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Conclusions

- Develop an analytical tool for performance evaluation of ILMMSE detection over MIMO channels
- Establish an area theorem for MIMO channels
- Prove that, with perfect CSIT, a properly designed LP-ILMMSE scheme can achieve the water-filling capacity for MIMO systems
- Propose an LP-ILMMSE scheme for MIMO systems without CSIT to approach the outage capacity within a marginal gap

Future Work

- Design of practical codes for the LP-ILMMSE scheme to approach the outage capacity

- Diversity and multiplexing tradeoff (DMT) analysis for the LP-ILMMSE scheme

- Design of LP-ILMMSE scheme for partial CSIT

Thank you !